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 $D \rightarrow K\ell\nu$  and  $D \rightarrow \pi\ell\nu$  form factors from Lattice QCDJ. KOPONEN<sup>1</sup>, C.T.H. DAVIES, G. DONALD

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We present a very high statistics study of  $D$  and  $D_s$  semileptonic decay form factors on the lattice. We work with MILC  $N_f = 2 + 1$  lattices and use the Highly Improved Staggered Quark action (HISQ) for both the charm and the strange and light valence quarks. We use both scalar and vector currents to determine the form factors  $f_0(q^2)$  and  $f_+(q^2)$  for a range of  $D$  and  $D_s$  semileptonic decays, including  $D \rightarrow \pi\ell\nu$  and  $D \rightarrow K\ell\nu$ . By using a phased boundary condition we are able to tune accurately to  $q^2 = 0$  and explore the whole  $q^2$  range allowed by kinematics. We can thus compare the shape in  $q^2$  to that from experiment and extract the CKM matrix element  $|V_{cs}|$ . We show that the form factors are insensitive to the spectator quark:  $D \rightarrow K\ell\nu$  and  $D_s \rightarrow \eta_s\ell\nu$  form factors are essentially the same, which is also true for  $D \rightarrow \pi\ell\nu$  and  $D_s \rightarrow K\ell\nu$  within 5%. This has important implications when considering the corresponding  $B/B_s$  processes.

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# 1 Introduction

Lattice QCD is an excellent tool for calculating strong interaction effects from first principles and can provide accurate phenomenology not available with any other method. Particularly important and interesting applications of this method are the determinations of various heavy meson weak decay matrix elements that are key to constraining the vertex of the Unitarity Triangle derived from the Cabibbo-Kobayashi-Maskawa (CKM) matrix. This provides a stringent test of the self-consistency of the Standard Model.

However, calculations of these matrix elements must not be seen in isolation. This is only one part of the HPQCD program to calculate meson spectra, decay constants and other QCD observables fully non-perturbatively (see e.g. the calculation of  $f_{D_s}$  in [1] or  $J/\psi$  mass, leptonic width and radiative decay rate to  $\eta_c$  in [2]). The Highly Improved Staggered Quark (HISQ) formalism enables us to keep the discretization errors small and to treat charm quarks the same way as light and strange quarks, which reduces the systematic errors.

A simple recipe for doing a Lattice QCD calculation is:

1. Generate sets of gluon fields from Monte Carlo integration of the QCD path integral (including effects of u, d and s sea quarks)
2. Calculate averaged hadron correlators from valence quark propagators
3. Fit the correlators as a function of time to obtain masses and matrix elements
4. Determine  $a$  and fix  $m_q$  to get results in physical units
5. Extrapolate to  $a = 0$  and physical light quark mass for real world

However, in this paper we don't go into details of Lattice QCD and concentrate on the results. The paper is organized as follows: In Section 2 we briefly explain how we calculate the form factors on the lattice and fit the results. We give our results for the form factors in Section 3 and talk about the  $z$ -expansion and continuum and chiral extrapolations in more detail in Section 4. In Section 5 we explain how we extract  $V_{cs}$  and give our result and summarise in Section 6.

## 2 Form factors on the lattice

Semileptonic form factors are 3-point amplitudes in Lattice QCD. To get information of both form factors,  $f_0(q^2)$  and  $f_+(q^2)$ , we calculate scalar and vector currents and the corresponding 2-point correlators for the mesons – see Fig. 1. The scalar current is

$$\langle K|S|D \rangle = f_0^{D \rightarrow K}(q^2) \frac{M_D^2 - M_K^2}{m_{0c} - m_{0s}}, \quad (1)$$

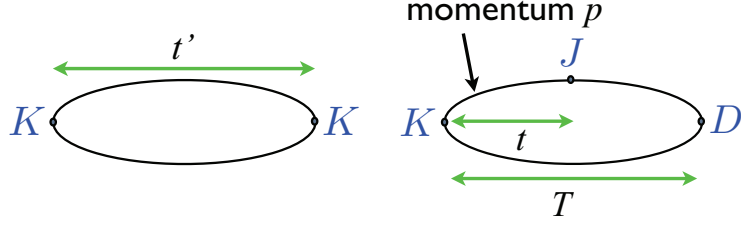


Figure 1: 2-point and 3-point correlators. In the 2-point correlator a kaon is created at time  $t'$  and annihilated at time 0. In the 3-point correlator the two mesons are time  $T$  apart and a scalar or vector current  $J$  is inserted at time  $t$  ( $0 < t < T$ ). One of the quarks can be given a momentum  $p$  to explore the full  $q^2$  range.

where  $q^\mu = p_D^\mu - p_K^\mu$  (using  $D \rightarrow K \ell \nu$  as an example), and the vector current can be written as

$$\langle K | V^\mu | D \rangle = f_+^{D \rightarrow K}(q^2) \left[ p_D^\mu + p_K^\mu - \frac{M_D^2 - M_K^2}{q^2} q^\mu \right] + f_0^{D \rightarrow K}(q^2) \frac{M_D^2 - M_K^2}{q^2} q^\mu. \quad (2)$$

Note that this guarantees that  $f_0(0) = f_+(0)$ . See also our earlier calculation of semileptonic form factors in [3]. We use 3 ensembles of MILC asqtad  $N_f = 2 + 1$  lattice configurations (sets 1, 2 and 4 in [2]).

From experiments we get the differential decay rates, e.g.

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 p_K^3}{24\pi^3} |V_{cs}|^2 |f_+^{D \rightarrow K}(q^2)|^2 \quad (3)$$

for  $D \rightarrow K \ell \nu$ . From this one can determine  $|V_{cs} \cdot f_+^{D \rightarrow K}(q^2)|$ , but one needs either  $f_+(q^2)$  from theory or  $V_{cs}$  from unitarity to determine the other.

## 2.1 Fitting the Lattice QCD results

We fit the lattice 2-point and 3-point correlators simultaneously, as a function of time  $t'$  and meson separation  $T$  (see Fig. 1) to estimate correlations between all fit parameters. We use multi-exponential fits (up to 5 exponentials) to reduce systematic errors from the excited states. The fit parameters are constrained by Bayesian priors. More details about the type of fits that we use can be found in [3].

We have very high statistics, of the order of 100000 correlators, which gives us very small statistical errors. By giving one of the quarks a momentum  $p$  using twisted boundary conditions [4], as shown in Fig. 1, we can tune accurately to  $q^2 = 0$  or choose any  $q^2$  value in the allowed kinematical region. This enables us to study the whole physical  $q^2$  range.

### 3 Semileptonic form factors

We have calculated the form factors  $f_0$  and  $f_+$ , as a function of  $q^2$ , for various  $D$  and  $D_s$  semileptonic decays. Let us divide the results into two groups: charm to strange decays and charm to light decays.

#### 3.1 Charm to strange decay: $D \rightarrow K\ell\nu$ , $D_s \rightarrow \eta_s\ell\nu$ and $D_s \rightarrow \phi\ell\nu$

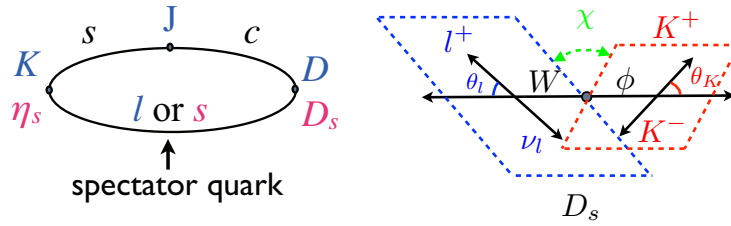


Figure 2: On the left:  $D \rightarrow K\ell\nu$  and  $D_s \rightarrow \eta_s\ell\nu$  decays are the same except for the spectator quark; On the right: Kinematics of the  $D_s \rightarrow \phi\ell\nu$  decay.

$D \rightarrow K\ell\nu$  and  $D_s \rightarrow \eta_s\ell\nu$  both have the same charm-strange current, as they are both charm to strange decays.  $\eta_s$  is the pseudoscalar  $s\bar{s}$  meson, so both decays are pseudoscalar to pseudoscalar decays. Note that  $\eta_s$  is not a physical meson, but can be easily calculated on a lattice. The difference between these two decays is the spectator quark, light vs. strange, as illustrated in Fig. 2.

Our results on different lattice ensembles are shown in Fig. 3, along with the final result after the continuum and chiral extrapolation. The  $q^2$  dependence is well understood qualitatively –  $f_+$  rises more steeply than  $f_0$ , as  $f_+$  is governed by the vector meson  $M_{D_s^*}$  in the pole mass parameterization, whereas  $f_0$  is governed by the scalar meson  $M_{D_{s0}^*}$ . The continuum and chiral extrapolation will be discussed in more detail in Section 4. The differences between the coarse and the fine lattice results are small, i.e. the discretisation effects are very well under control.

Note that the shapes of the form factors do not depend on the spectator quark: the form factors for these two decays,  $D \rightarrow K\ell\nu$  and  $D_s \rightarrow \eta_s\ell\nu$ , are the same within 3% and even closer when one moves away from  $q^2 = 0$ . Comparing the decay constants of the two mesons,  $f_D$  and  $f_{D_s}$ , one might expect a change of about 15% when going from a light spectator quark to a strange quark. However, this does not appear to be the case for form factors.

On the lattice one can also calculate form factors for a pseudoscalar to a vector meson semileptonic decay.  $D_s \rightarrow \phi\ell\nu$  is a charm to strange decay like  $D \rightarrow K\ell\nu$  and

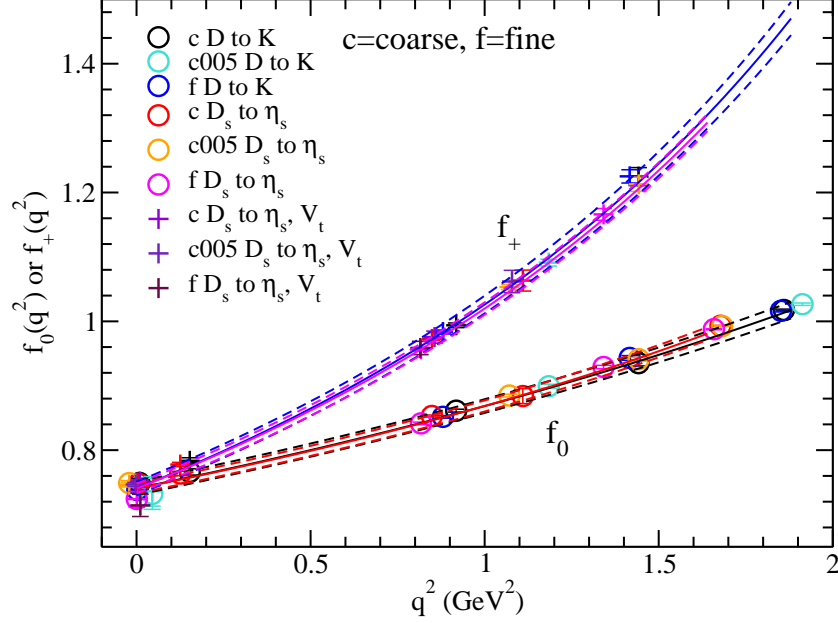


Figure 3:  $D \rightarrow K \ell \nu$  and  $D_s \rightarrow \eta_s \ell \nu$  form factors. The form factors are very insensitive to the spectator quark.

$D_s \rightarrow \eta_s \ell \nu$ . As  $\phi$  is a vector meson, there are now more form factors than in the pseudoscalar to pseudoscalar decay. The general form for a pseudoscalar to vector matrix element is

$$\begin{aligned} \langle \phi(p', \epsilon) | V^\mu - A^\mu | D_s(p) \rangle = & \frac{2i\epsilon_{\mu\nu\alpha\beta}}{M_{D_s} + M_\phi} \epsilon^\nu p_{D_s}^\alpha p_\phi^\beta V(q^2) - (M_{D_s} + M_\phi) \epsilon^\mu A_1(q^2) \\ & + \frac{\epsilon \cdot q}{M_{D_s} + M_\phi} (p + p') A_2(q^2) + 2M_\phi \frac{\epsilon \cdot q}{q^2} q^\mu A_3(q^2) - 2M_\phi \frac{\epsilon \cdot q}{q^2} q^\mu A_0(q^2), \end{aligned} \quad (4)$$

where

$$A_3(q^2) = \frac{M_{D_s} + M_\phi}{2M_\phi} A_1(q^2) - \frac{M_{D_s} - M_\phi}{2M_\phi} A_2(q^2) \text{ and } A_3(0) = A_0(0). \quad (5)$$

Here  $p$  is the momentum of the  $D_s$ ,  $p'$  is the momentum of the  $\phi$  and  $\epsilon$  is the polarisation vector.  $V(q^2)$  is the vector form factor and  $A_0(q^2)$ ,  $A_1(q^2)$ ,  $A_2(q^2)$ ,  $A_3(q^2)$  are axial vector form factors. Note that only three of the axial vector form factors are independent. We can extract the different form factors by choosing the right kinematics – see [5] for more details. The diagram on the right in Fig. 2 shows the kinematics of this decay:  $\theta_e$  is the angle between the  $e^+$  and the  $W$  and  $\theta_K$  is the angle between the kaon and the  $\phi$ .  $\theta_e$  is measured in the rest frame of the  $W$  and  $\theta_K$  is measured in

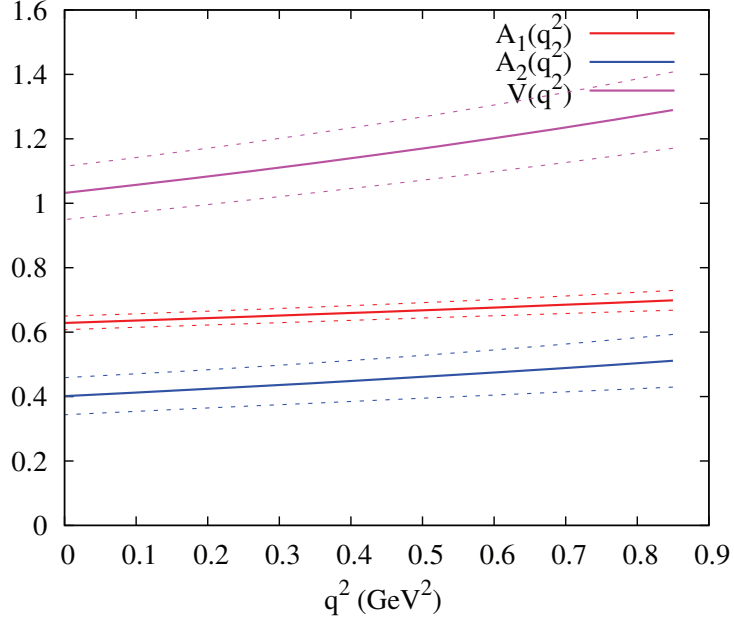


Figure 4:  $D_s \rightarrow \phi \ell \nu$  form factors, defined in [5].

the rest frame of the  $\phi$ . Our results, the form factors and angular distributions, are plotted in Figs. 4 and 5. The ‘boxes’ in Fig. 5 show the experimental results from BaBar [8] for the angular distributions – we see good agreement between theory and experiment.

### 3.2 Charm to light decay: $D \rightarrow \pi \ell \nu$ and $D_s \rightarrow K \ell \nu$

We calculate two charm to light semileptonic decays,  $D \rightarrow \pi \ell \nu$  and  $D_s \rightarrow K \ell \nu$ . In this case, both of the decays are experimentally accessible. Our results on different lattice ensembles, along with the final result after the continuum and chiral extrapolation (see Section 4), are shown in Fig. 6. The conclusions are very similar to the charm to strange decay: Again, the dependence of the form factors on the spectator quark mass is very mild – going from a strange spectator quark to a light quark changes the form factors by less than 5%.

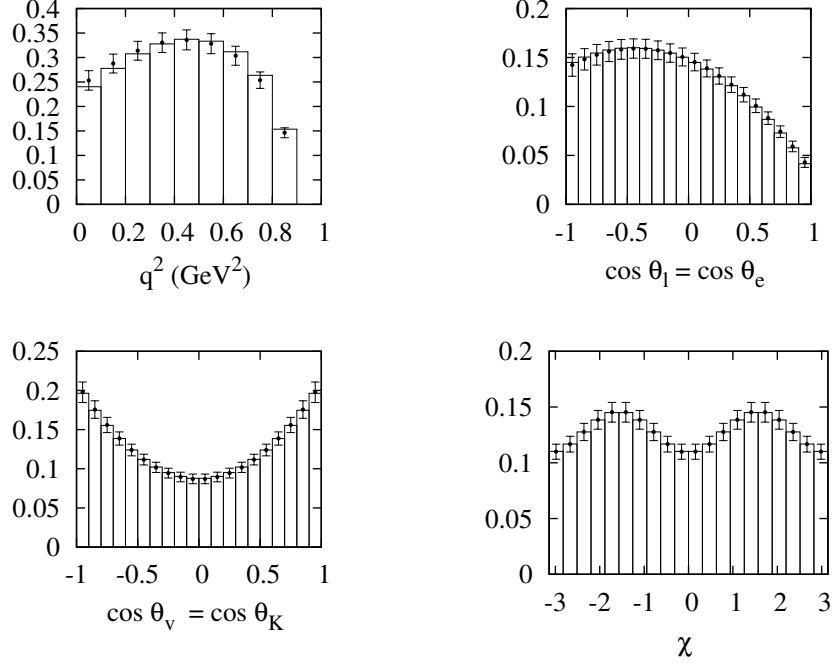


Figure 5:  $D_s \rightarrow \phi \ell \nu$  decay rates in  $q^2$  bins and angular distributions. The angles are defined in Fig. 2.

## 4 The $z$ -expansion

For continuum and chiral extrapolations we use the  $z$ -expansion [6]: First we remove the poles (using  $D \rightarrow K$  decay here as an example)

$$\tilde{f}_0^{D \rightarrow K}(q^2) = \left(1 - \frac{q^2}{M_{D_{s0}^*}^2}\right) f_0^{D \rightarrow K}(q^2), \quad \tilde{f}_+^{D \rightarrow K}(q^2) = \left(1 - \frac{q^2}{M_{D_s^*}^2}\right) f_+^{D \rightarrow K}(q^2) \quad (6)$$

and convert to  $z$  space

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+}}{\sqrt{t_+ - q^2} + \sqrt{t_+}}, \quad t_+ = (M_D + M_K)^2. \quad (7)$$

The transformation of the complex  $q^2$  plane to the  $z$  plane is sketched in Fig. 7. This is useful, as the form factors  $f$  in the physical region can be described by a simple power series in  $z$ :

$$\tilde{f}_0^{D \rightarrow K}(z) = \sum_{n \geq 0} b_n(a) z^n, \quad \tilde{f}_+^{D \rightarrow K}(z) = \sum_{n \geq 0} c_n(a) z^n, \quad c_0 = b_0. \quad (8)$$

We let the fit parameters  $b_n, c_n$  depend on lattice spacing and sea quark masses. In the end we take  $a = 0$  and  $m_q = m_q^{\text{phys}}$  to get the result in the physical limit. Fig. 7 shows one of the fits, for  $D \rightarrow K \ell \nu$ .

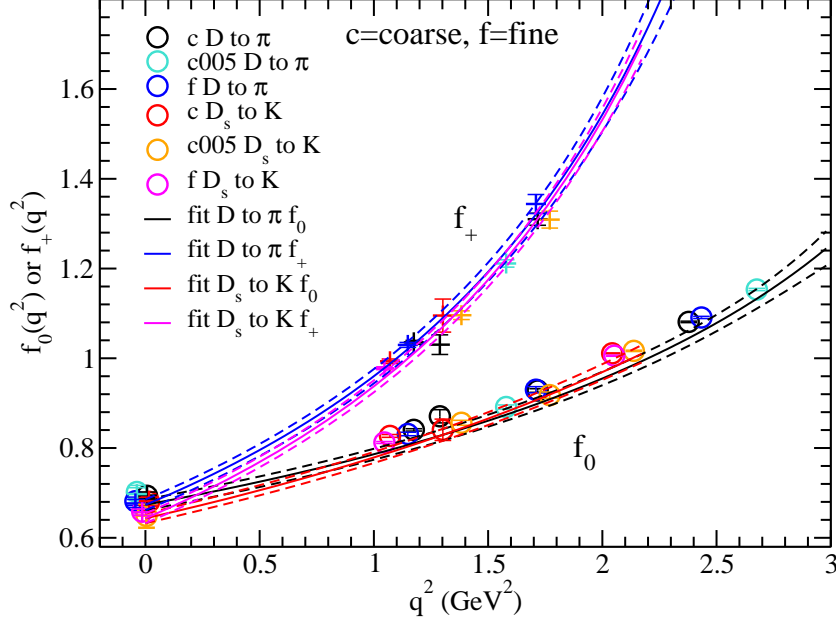


Figure 6:  $D \rightarrow \pi \ell \nu$  and  $D_s \rightarrow K \ell \nu$  form factors. The form factors are not sensitive to the spectator quark.

## 5 Extracting $V_{cs}$

One of the goals is to determine the CKM matrix elements and over-constrain the unitarity triangle by providing accurate input from theory. The lattice calculation does not know about  $V_{cs}$ , but experiments do. By combining the lattice calculation of the  $D \rightarrow K \ell \nu$  form factor  $f_+(q^2)$  with experimental results we can extract  $V_{cs}$  without the need to assume unitarity of the CKM matrix. By integrating the lattice form factors over the experimental  $q^2$  bins, i.e. integrating Eq. 3 to get the rate for a given bin and then taking the experiment to lattice ratio gives us  $V_{cs}^2$  for that given bin. Our results are shown in Fig. 8. We do this using CLEO [7], BaBar [8], Belle [9] and BESIII (preliminary, [10]) results and fit a constant to these  $V_{cs}^2$  values, including bin to bin correlations from lattice calculations and experiments in the fit. Including different sets of experimental results does not change our value for  $V_{cs}$ , as can be seen in Fig. 9. Our best, preliminary value is  $V_{cs} = 0.965(14)$ . This is consistent with  $V_{cs} = 0.97344(16)$  from PDG [12], that is calculated by assuming unitarity. Using our best value for  $V_{cs}$  we plot the  $D \rightarrow K \ell \nu$  decay rates in  $q^2$  bins in Fig. 10, which shows excellent agreement between theory and experiments.  $D_s \rightarrow \phi \ell \nu$  decay rates, plotted in Fig. 5, also show excellent agreement.



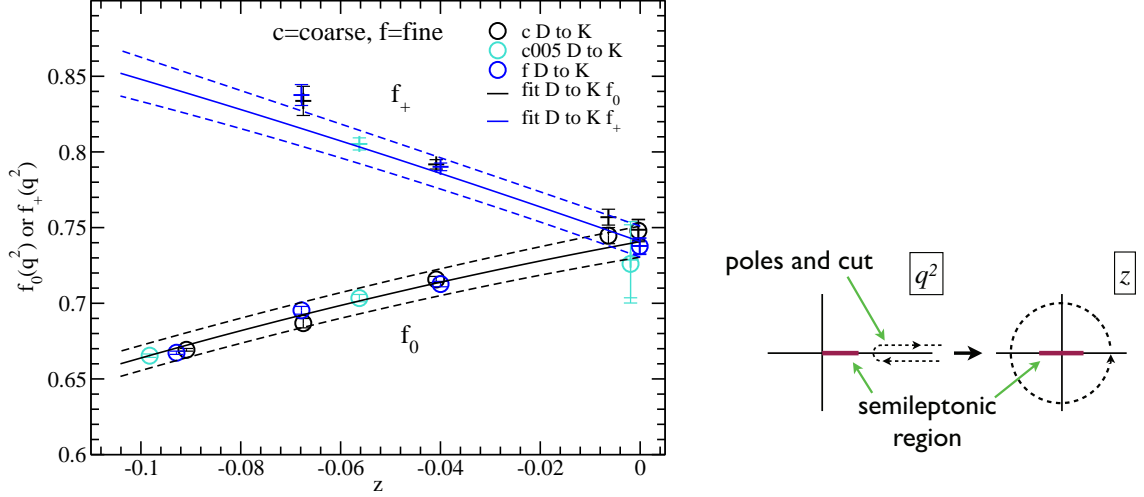


Figure 7: On the left:  $D \rightarrow K\ell\nu$  form factors in  $z$  space and the fit. On the right: Transformation of the complex  $q^2$  plane to the  $z$  plane.

## 6 Summary

This very high precision Lattice QCD calculation provides  $D$  and  $D_s$  meson semileptonic decay form factors from first principles. We study the full  $q^2$  range and different semileptonic decays (different daughter mesons). We determine the form factors to better than 3% accuracy (better than 2% in the case of  $D \rightarrow K\ell\nu$ ). The  $D/D_s$  form factors are very insensitive to the spectator quark and this is expected to be true for  $B/B_s$  as well. We calculate decay rates in  $q^2$  bins to compare with experiments and extract  $V_{cs}$ . We get  $V_{cs} = 0.965(14)$  (preliminary), which gives very good agreement with experiments, i.e. the shape of the form factor  $f_+^{D \rightarrow K}$  calculated in Lattice QCD agrees with experimental results.

## ACKNOWLEDGEMENTS

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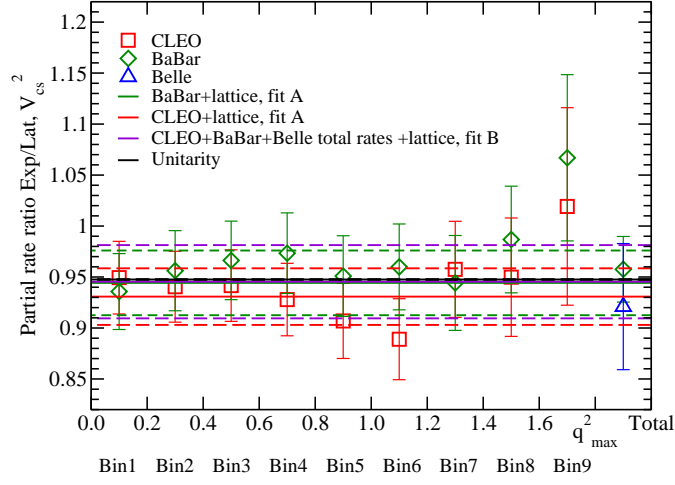


Figure 8: Ratio of experimental to lattice results in each  $q^2$  bin for  $D \rightarrow K\ell\nu$ , i.e.  $|V_{cs}|^2$  extracted from that bin directly. The experimental results are from [7, 8, 9, 10].

## References

- [1] HPQCD Collaboration; C. T. H. Davies, C. McNeile, E. Follana, G. P. Lepage, H. Na, and J. Shigemitsu, PRD 82, 114504 (2010)
- [2] HPQCD Collaboration; G. C. Donald, C. T. H. Davies, R. J. Dowdall, E. Follana, K. Hornbostel, J. Koponen, G. P. Lepage and C. McNeile, arXiv:1208.2855
- [3] HPQCD Collaboration; H. Na, C. T. H. Davies, E. Follana, G. P. Lepage and J. Shigemitsu, PRD 82, 114506 (2010)
- [4] D. Guadagnoli, F. Mescia and S. Simula, PRD 73, 114504 (2006)
- [5] HPQCD; G. Donald, C. T. H. Davies and J. Koponen, PoS(Lattice 2011)278
- [6] C. Bourrely, I. Caprini and L. Lellouch, PRD 79, 013008 (2009)
- [7] CLEO Collaboration, PRD 80, 032005 (2009)
- [8] BaBar Collaboration, PRD 76, 052005 (2007) and PRD 78, 051101(R) (2008)
- [9] Belle Collaboration, PRL 97, 061804 (2006)
- [10] C. Liu, The 5th International Workshop on Charm Physics, arXiv:1207.1171
- [11] A. Zupanc's talk in the 5th International Workshop on Charm Physics
- [12] J. Beringer et al. (Particle Data Group), PRD 86, 010001 (2012).

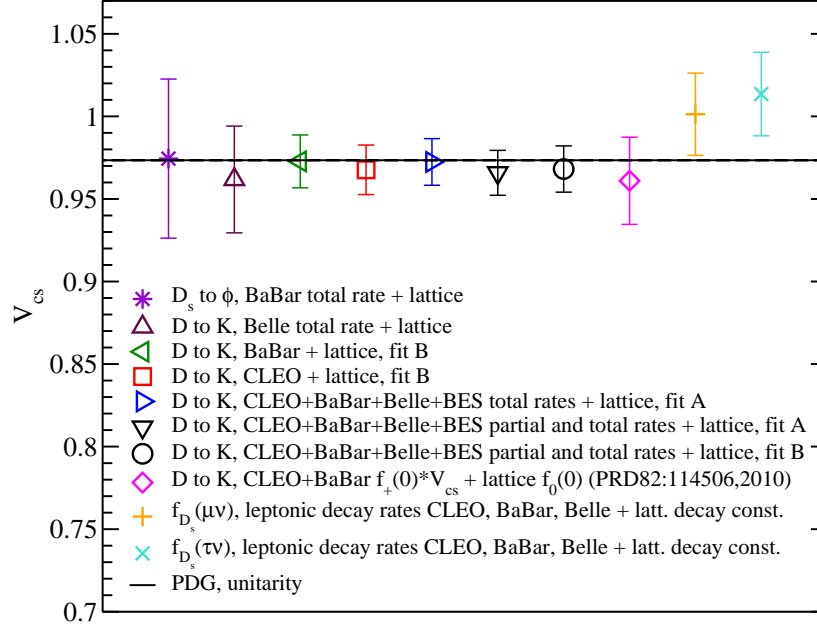


Figure 9: Our determination of  $V_{cs}$  from this Lattice QCD study combined with various sets of experimental results.  $D \rightarrow K\ell\nu$  experimental results are from [7, 8, 9, 10]. The decay constant  $f_{D_s}$  is from [1] and leptonic decay rates are from [11].

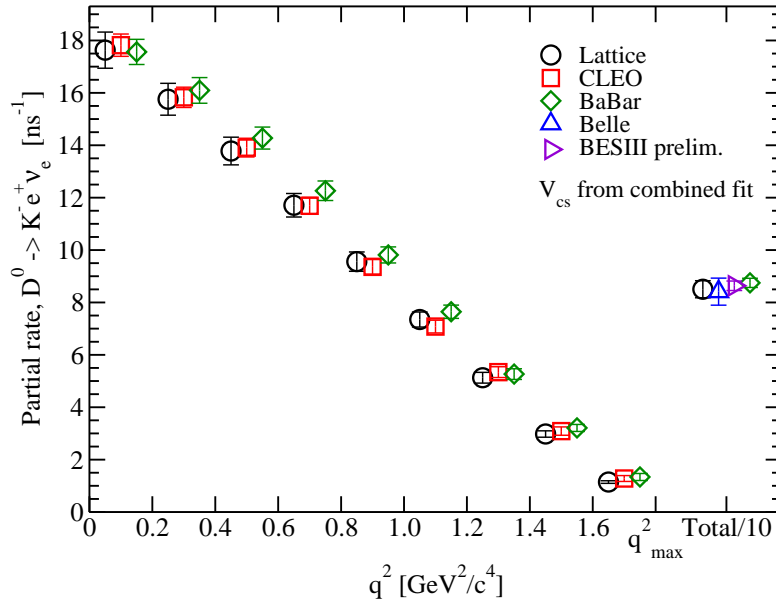


Figure 10: Decay rates in  $q^2$  bins for  $D \rightarrow K\ell\nu$ . The lattice numbers include our result for  $V_{cs}$ . The experimental results are from [7, 8, 9, 10].